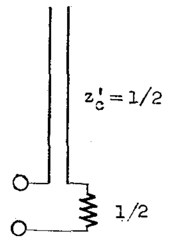

Fig. 1

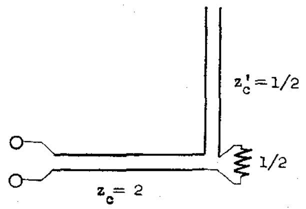

Fig. 2

It is apparent that this third condition is sufficient and mathematical induction will readily prove that it is necessary.

This additional restriction completes the necessary and sufficient conditions. This theorem may also be easily reduced from the writer's theorem 1 presented in anothe publication.[2]

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[2] H. Ozaki and J. Ishii, "Synthesis of transmission-line networks and the design of UHF filters," IRE Trans. on Circuit Theory, vol. CT-2, pp. 325–336; December, 1955. See theorem 1, p. 326.

Ferrite Directional Couplers with Off-Center Apertures*

Introduction

A recent study[1,2] of Bethe's small-hole coupling theory has led to an extension of his work to include the case where the coupling aperture is filled with an anisotropic ferrite. This new theory is applicable to any situation where Bethe's coupling theory is useful and is hindered chiefly by inadequate expressions for the magnetic dipole moments of the ferrite. Fortunately, simple expressions[3] for these magnetic dipole moments are available when the ferrite sample is small compared with the wavelength inside it. Experimental verification of this new coupling theory was obtained with a cross-guide coupler and with a collinear coupler for a centered coupling hole.[4] Since the new theory is equally applicable to situations where the coupling is off center,[5] this paper considers the theoretical and experimental aspects more fully. This seems especially worthwhile since other workers[6,7] have recently considered the case of an off-center aperture. However, their work was performed from either a different viewpoint or else less rigorously. The new coupling theory is quite general and can be used irrespective of waveguide configuration or propagating mode.

In this paper, simple expressions for the coupling and directivity of a cross-guide coupler and a collinear coupler will be presented with some experimental results. In all cases, the ferrite parameters were chosen to obtain a good correspondence between the theoretical and experimental curves of coupled power. Unfortunately, the sample size used was too large to expect exact agreement between theory and experiment.

Theoretical Results

The following theoretical expressions for the coupling and the directivity are good approximations when the coupling hole is small compared with wavelength and when the ferrite sample is small compared with the wavelength inside it. No attempt has been made to determine the limits of the validity of the approximate expressions although experiments indicate that the theory governing the behavior of the ferrite is too simplified for many applications.

The amplitudes of the normal modes excited in the secondary waveguide by a unit normal mode in the primary waveguide are given elsewhere for both the cross-guide coupler[8] and the collinear coupler.[9] The expressions we use below for the coupled power are valid only for two identical rectangular waveguides propagating the TE_{10} mode and for a round coupling hole of diameter d. The coupling hole location is given by x and ξ for $0 \leq x \leq a$ and for $0 \leq \xi \leq a$, where a is the width of the waveguide. The orientation of the waveguides and the definition of the three sets of axes are given in Fig. 1.

For the cross-guide coupler, let us consider the expression for the power coupled in both directions in the secondary waveguide when the coupling hole is located at a point of circular polarization; *i.e.*, when

$$\tan \frac{\pi x}{a} = \tan \frac{\pi \xi}{a} = \lambda_g/\lambda_c.$$

For this case, let us choose the frequency so that $\lambda_g = \lambda_c$. Thus, we obtain in db

$$C_{\perp}^{\pm} = C_0 + 20 \log \left| j(1-\chi_{ll})\Upsilon + R\Psi + \chi_{lm}\Omega \right| \quad (1)$$

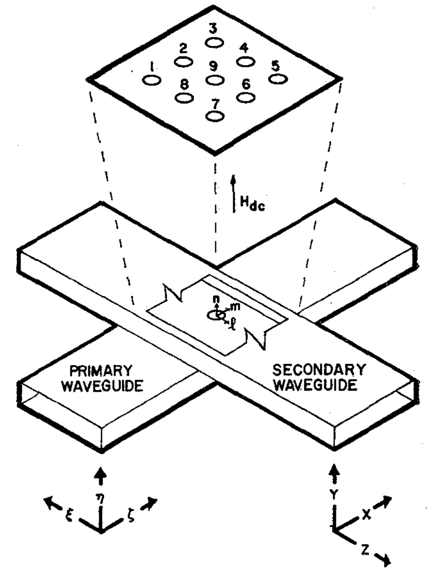

Fig. 1—Cross-guide directional coupler.

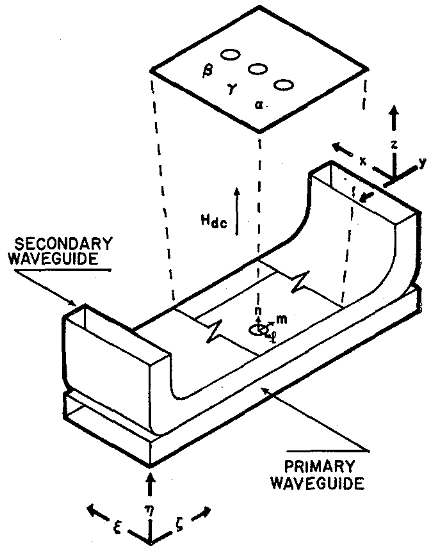

Fig. 2—Collinear directional coupler.

where

$$\Upsilon = \sin \frac{\pi}{a} (\xi \pm x)$$

$$\Psi = \sin (\pi x)/a \sin (\pi \xi)/a$$

and

$$\Omega = \cos \frac{\pi}{a} (\xi \mp x).$$

The upper and lower signs in the superscript indicate the power coupled in the positive and negative directions, respectively. All of the symbols used are defined elsewhere[10] except R which is defined as $R = QF_E F_H^{-1}$.

For the collinear coupler, the expressions under the same conditions are the following in db:

$$C_{\parallel}^{\pm} = C_0 + 20 \log \left| -(1-\chi_{ll})\Omega + j\chi_{lm}\Upsilon + R\Psi \right| \quad (2)$$

where the orientation of the axes is defined in Fig. 2. Eqs. (1) and (2) are given a further

* Received by the PGMTT, March 24, 1958. Presented at the 1957 Annual PGMTT Meeting, New York, N. Y., May 9, 1957.

[1] D. C. Stinson, "Coupling through an aperture containing ferrites," Ph.D. dissertation, Dept. of Elec. Eng., University of California, Berkeley, Calif.; 1956.

[2] D. C. Stinson, "Coupling through an aperture containing an anisotropic ferrite," IRE Trans. on Microwave Theory and Techniques, vol. MTT-5, pp. 184–191; July, 1957.

[3] *Ibid.*, see Appendix.

[4] *Ibid.*, pp. 188–189.

[5] *Op. cit.*, Ph.D. dissertation, Appendix E.

[6] R. W. Damon, "Magnetically controlled microwave directional coupler," *J. Appl. Phys.*, vol. 26, pp. 1281–1283; October, 1955.

[7] A. D. Berk and E. Strumwasser, "Ferrite directional couplers," Proc. IRE, vol. 44, pp. 1439–1446; October, 1956.

[8] Stinson, *op. cit.*, "Coupling through an aperture containing an anisotropic ferrite," (16).

[9] *Ibid.*, (12).

[10] *Ibid.*, (22) and (28).

TABLE I

COUPLING FOR VARIOUS APERTURE LOCATIONS

Large		Small		Constant	
$C_1^-(+)$	$C_2^{\pm}(+)$	$C_1^-(-)$	$C_2^+(-)$	$C_3^-(\pm)$	$C_1^+(\pm)$
$C_3^+(+)$	$C_4^+(+)$	$C_4^+(-)$	$C_4^-(+)$	$C_\alpha^-(\mp)$	$C_\beta^-(\pm)$
$C_5^-(-)$	$C_6^{\pm}(-)$	$C_8^-(-)$	$C_8^+(+)$		
$C_7^+(-)$	$C_8^+(-)$	$C_\alpha^+(+)$	$C_\beta^+(-)$		
$C_4^-(-)$	$C_8^-(+)$	$C_\gamma^+(+)$	$C_\gamma^-(-)$		
$C_\alpha^+(-)$	$C_\beta^+(+)$				

variation since the susceptibility χ_{lm} is an odd function of the applied magnetostatic field.

The directivity is defined as the ratio in decibels of the power coupled in a particular direction in the secondary arm for an incident wave in the forward and backward directions, respectively, in the primary arm. Thus, we obtain in db

$$D = |C^+ - C^-|. \quad (3)$$

The coupling defined here is the negative of the standard definition. This is done so that an increase in coupled power corresponds to an increase in the ordinate of the curve of coupling vs magnetostatic field or frequency.

It is interesting to note, by inspection of (1) or (2), that the power coupled in one direction can be made independent of the magnetic susceptibilities by a proper choice of the hole location. Moreover, the susceptibilities can be made much larger than unity and can be controlled by the applied magnetostatic field. For a spherical ferrite sample, the maximum value of the susceptibility is inversely proportional to the reduced damping constant.[11] For ordinary microwave ferrites, this results in a maximum increase in coupling of about 20 db. The aperture locations in the cross-guide coupler and in the collinear coupler for which the coupling is significantly affected by the magnetostatic field are shown in Figs. 1 and 2, respectively. Information concerning the coupled power for some of the various aperture locations is given in Table I. The notation $C_i^-(+)$ means the coupling in the negative direction, aperture location i, and a positive value of the applied magnetostatic field. Thus, the sense of the magnetostatic field is indicated by the plus or minus sign inside the parentheses.

EXPERIMENTAL RESULTS

Curves of coupling vs the magnitude of the magnetostatic field for the cross-guide coupler, aperture location 1, and a Ferramic R-1 sphere are given in Fig. 3(a). Using (3), the directivity of the coupler for a positive magnetostatic field of 3 kilo-oersteds is 26 db. The coupling is −37 db. Curves of coupling vs the magnitude of the magnetostatic field for the collinear coupler, aperture location α and a Ferramic R-1 sphere are given in Fig. 3(b). The directivity for a negative magnetostatic field of 3 kilo-oersteds is 27 db. The coupling is −37 db. Another aperture location is γ, the centered hole. The coupling curves for this are shown in Fig. 3(c). An interesting characteristic is that the directivity is about 26 db with no

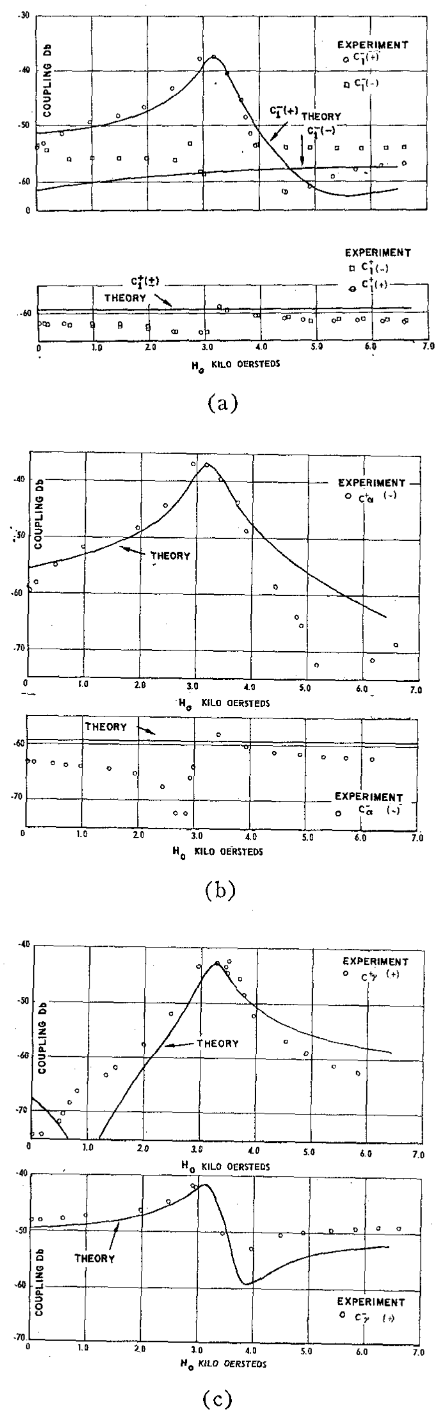

Fig. 3—Comparison of theory and experiment for coupling with Ferramic R-1 sphere of 0.124-inch diameter. Theoretical values for $\lambda_1 = 0.08$, $R = 1.278$, $E = 0.813$, $C' = 0.816$: (a) cross-guide coupler, aperture location 1; (b) collinear coupler, aperture location α; (c) collinear coupler, aperture location γ.

applied magnetostatic field which decreases to zero when the magnetostatic field increases to 3 kilo-oersteds. In the cases considered, agreement between theory and experiment is acceptable in a qualitative sense. Unfortunately, the sphere was too large to expect exact quantitative agreement. Coupling curves were also run on a 3-hole coupler using 3 Ferramic R-1 spheres and aperture location α. The center hole was larger than the end holes in order to improve the directivity. The measured coupling curves were similar to those in Fig. 3(b). For a negative applied magnetostatic field of 3 kilo-oersteds, the directivity was 42 db and the coupling was −26 db. For no magnetostatic field, the coupling was −46 db and the directivity was at least 30 db.

CONCLUSIONS

Ferrite directional couplers display several advantages over normal directional couplers, such as nonreciprocal coupling and the electrical control of coupling. The couplers also permit one to obtain fairly good absolute data and quite good comparative data on the characteristics of ferrite materials. Work in this area will be reported at a later date.

ACKNOWLEDGMENT

The author wishes to thank M. L. Fisher for considerable aid in the measurement program.

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[11] *Ibid.*, see Appendix.

A Modulator for Microwave Mixers*

A method is described for producing an amplitude modulated wave at an intermediate frequency from the mixing of two cw signals in a coaxial or waveguide system. The method is adaptable to any frequency range in which crystal mixers are used. At least 82 per cent modulation is produced by this method, and the envelope is a square wave. The repetition rate is 0 to 20 kc with presently available commercial components.

One advantage of this system is to provide an ac signal for further amplification. In addition, the frequency and phase of the envelope depends only on the stability of the audio generator and the chopper. This makes the system suitable for use with a phase sensitive detector. Thus, the bandwidth of the detector system can be readily reduced to 1 cps. This system of modulation has been used to replace the swept local oscillator in a microwave attenuation measurement system.

Fig. 1 is a wiring diagram of the modulator. As is well known, the impedance and conversion loss of a crystal depend on the

* Received by the PGMTT, March 19, 1958.